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DEVELOPMENT OF GAUSSIAN DIOPHANTINE QUADRUPLES WITH PROPERTY $D(16k^2)$

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ABSRACT
We construct Gaussian Diophantine quadruples with the property $D(16k^2)$.

KEYWORDS: - Perfect Square, Gaussian Diophantine Quadruples, Gaussian Integer.

INTRODUCTION

Many mathematicians analyzed the existence of a Diophantine triples and a Diophantine quadruples with property $D(t)$[1-4], where t is either an integer or any linear polynomial in t. A complex Diophantine m - tuple with property $D(z)$ is a set of m Gaussian integer, if the multiplication of any two different Gaussian integers increased by z is a perfect square. Various authors constructed the complex Diophantine quadruples in [5-10].
In this communication, Gaussian Diophantine quadruples with the property $D(16k^2)$ are constructed.

METHOD OF ANALYSIS

Consider two Gaussian integers $p = ka + kib + k$ and $q = ka + kib - 7k$
Using above Gaussian integers we observe that

$$pq + 16k^2 = (ka - 3k + kib)^2 = u^2$$

Here we attained Gaussian Diophantine double (p, q) with the property $D(16k^2)$.
Let r be any non-zero Gaussian integer such that

$$pr + 16k^2 = v^2 \tag{1}$$

$$qr + 16k^2 = w^2 \tag{2}$$

Applying the linear transformation $v = p + u, \quad w = q + u$ and subtracting (1) from (2) gives the value of r as

$$r = p + q + 2u$$

Hence we get

$$r = 4ka - 12k + 4kib$$

Applying the formula $s = p + q + r + \frac{2}{t}[pqr + uvw]$ for finding the fourth tuple, we attain

$$s = ka^3 - 9ka^2 + 23ka - 8kab^2 + 9kb^2 - 15k + i(3ka^2b - 18kab + 23kb - kb^3)$$

Hence, we obtained the Gaussian Diophantine quadruples

$$\begin{pmatrix} ka + kib + k,\ ka + kib - 7k,\ 4ka - 12k + 4kib, \\ ka^3 - 9ka^2 + 23ka - 8kab^2 + 9kb^2 - 15k + i(3ka^2b - 18kab + 23kb - kb^3) \end{pmatrix}$$

Note:
Suppose we consider the linear transformation, $v = p - u, \quad w = q - u$ and applying the same procedure mentioned above, we obtain $r = 0$ and $s = 0$.

Thus, we get a trivial Gaussian Diophantine quadruples $(ka+kib+k, ka+kib-7k, 0, 0)$ with the same property mentioned above.

CONCLUSION

In this paper, Gaussian Diophantine quadruples with the property $D(16k^2)$ are constructed. One may search for some other Gaussian Diophantine quadruples involving special numbers with the suitable property.

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